Conference

"SPECTRAL AND ANALYTIC INVERSE PROBLEMS"

IHP, May 04 -07

Organisers : David Dos Santos Ferreira, Luc Robbiano and Thierry Daudé

Scientific Committee : Yaroslav Kurylev, Vesselin Petkov, Mikko Salo, Gunther Uhlmann, Steve Zelditch.

PROGRAM

Monday, May 04

9 :00 - 9 :30 : Registration

9 :30 - 10 :30 : **John Sylvester** (University of Washington).
The Role of Evanescence in the Inverse Source Problem

10 :30 - 11 :00 : Coffee, tea, croissants.

11 :00 - 12 :00 : **David Colton** (University of Delaware).
Transmission Eigenvalues for Spherically Stratified Media

12h00-14h00 : *Lunch.*

14 :00 - 15 :00 : **Evgeny Lakshtanov** (University of Aveiro).
On some constructive and explicit solutions in inverse scattering theory

15 :00 - 16 :00 : **Eemeli Blåsten** (University of Helsinki).
A new viewpoint to scattering theory à la Hörmander

16 :00 - 16 :30 : Coffee, tea, cookies.

16 :30 - 17 :30 : **Ting Zhou** (Northeastern University).
On uniqueness of an inverse problem for the time-harmonic Maxwell equations

Tuesday, May 05

9 :00 - 10 :00 : **Hiroshi Isozaki** (Tsukuba University).
Inverse scattering on non-compact manifolds with general metric

10 :00 - 10 :30 : Coffee, tea, croissants.

10 :30 - 11 :30 : **Tanya Christiansen** (University of Missouri).
Resonances for Schrödinger operators

11 :30 - 12 :30 : **Antonio Sà Barreto** (Purdue University).
Unique continuation from infinity and inverse problems in geometric scattering.

12h30-14h30 : *Lunch.*

14 :30 - 15 :30 : **Eric Soccorsi** (Université de Marseille).
An inverse spectral problem in a periodic quantum waveguide

15 :30 - 16 :30 : **Yavar Kian** (Université de Marseille).
A multidimensional Borg-Levinson theorem for magnetic Schrödinger operators with partial spectral data

16 :30 - 17 :00 : Coffee, tea, cookies.

17 :00 - 18 :00 : **Andras Vasy** (Stanford University).
The local inverse problem for the geodesic X-ray transform on tensors and boundary rigidity

Wednesday, May 06

9 :00 - 10 :00 : **Georgi Popov** (Université de Nantes).
Isospectral deformations K.A.M. tori and spectral rigidity

10 :00 - 10 :30 : Coffee, tea, croissants.

10 :30 - 11 :30 : **Gerald Teschl** (University of Vienna).
Peakon asymptotics for the dispersionless Camassa-Holm equation

11 :30 - 12 :30 : **Yves Colin de Verdière** (Institut Fourier, Grenoble).
Direct and inverse spectral problems for sub-Riemannian (sR) Laplacians

12h30-14h30 : *Lunch.*

14 :30 - 15 :30 : **Mirren Zubeldia** (University of Helsinki).
Transmission eigenvalues for magnetic Schrödinger operator

15 :30 - 16 :30 : **Hongyu Liu** (Hong Kong Baptist University).
Stable determination of polyhedral scatterers by a minimal number of far-field measurements

16 :30 - 17 :00 : Coffee, tea, cookies.

17 :00 - 18 :00 : **Alberto Ruiz** (Universidad Autónoma de Madrid).
Stability of the Calderón problem with partial data

18 :30 :

COCKTAIL DINATOIRE

Thursday, May 07

9 :00 - 10 :00 : **Boaz Haberman** (University of California, Berkeley).
Unique continuation and inverse boundary value problems for elliptic operators with unbounded gradient terms.

10 :00 - 10 :30 : Coffee, tea, croissants.

10 :30 - 11 :30 : **Rakesh** (University of Delaware).
The inverse backscattering problem

11 :30 - 12 :30 : **Laurent Bourgeois** (ENSTA).
The inverse Robin problem revisited for non-smooth coefficient

12h30-14h30 : *Lunch.*

14 :30 - 15 :30 : **Alexandre Jollivet** (Université de Lille).
Inverse scattering at high energies for classical particles in a long range electromagnetic field

15 :30 - 16 :30 : **Allan Greenleaf** (University of Rochester).
Virtual Hybrid Edge Detection

16 :30 - 17 :00 : Coffee, tea, cookies.

Abstracts :

Eemeli Blåsten (University of Helsinki).

A new viewpoint to scattering theory à la Hörmander

Abstract : I will describe a new method to construct fundamental solutions to constant coefficient PDEs. They behave well enough to generate scattering solutions. This is a work in progress with John Sylvester. The method generalizes scattering theory to operators that are not simply characteristic in the sense of Agmon and Hörmander. I will start with a short description of scattering theory based on research by these two men and then describe our result and method which is based on the simple situation of 1D scattering.

Laurent Bourgeois (ENSTA).

The inverse Robin problem revisited for non-smooth coefficient

Abstract : In this talk we revisit the classical inverse Robin problem, which consists in finding the ratio between the normal derivative and the trace of the solution (the Robin coefficient) on a subset of the boundary, given the Cauchy data (both the normal derivative and the trace of the solution) on the complementary subset. We are interested in the uniqueness problem for a Robin coefficient which is merely in L^∞ and a Neumann data in L^2. We first prove uniqueness of the Robin coefficient for a problem governed by the Laplace equation in a Dini-smooth domain in 2D, and give some indications about the generalization of such result in a Lipschitz domain and for a problem governed by a conductivity equation with a conductivity chosen in $W^{1,r}$, where $r > 2$. We also prove a non-uniqueness result in the $3D$ case. Eventually, as a simple consequence of uniqueness in $2D$, we prove a Lipschitz stability estimate when the Robin coefficient is supposed to lie in a finite dimensional space.

This is a joint work with Laurent Baratchart and Juliette Leblond.

Tanya Christiansen (University of Missouri).

Resonances for Schrödinger operators

Abstract : Mathematically, resonances may serve as an analog of eigenvalues for a class of operators on noncompact domains. Physically, they correspond to decaying waves, or metastable states.

Consider a Schrödinger operator $-\Delta + V$ on $\mathbf{R}l^d$, with $V \in L_0^\infty(\mathbf{R}^d)$. The resonances lie on different spaces depending on the parity of the dimension d. We give some lower bounds on resonance counting functions, particularly highlighting the case of even dimension d and differences with the odd-dimensional case. Moreover, we point out that some of the proofs used to obtain lower bounds

on resonance counting functions can, with relatively simple modifications, give further inverse-type results.

Yves Colin de Verdière (Institut Fourier, Grenoble).

Direct and inverse spectral problems for sub-Riemannian (sR) Laplacians

Abstract : In a joint work in progress with Emmanuel Trélat and Luc Hillairet, we prove a Quantum Ergodicity theorem for some sR Laplacians. I will present these results as well as results and conjectures about inverse spectral problems for sR Laplacians.

David Colton (University of Delaware).

Transmission Eigenvalues for Spherically Stratified Media

Abstract : The transmission eigenvalue problem plays a central role in the qualitative approach to inverse scattering theory (c.f. F. Cakoni and D. Colton, A Qualitative Approach to Inverse Scattering Theory, Springer, 2014). In this talk we consider the transmission eigenvalue problem for spherically stratified media such that the eigenfunctions are also spherically stratified and, in particular, the inverse spectral problem for this case. Even this simple case has a surprisingly rich analytic theory. For example, we will show that complex eigenvalues exist in general (but not always!) and under certain conditions lie in a strip parallel to the real axis (but under certain conditions do not lie in a strip!). Our analysis is based on the theory of entire functions of a complex variable. This is joint work with Y.J. Leung and S. Meng.

Allan Greenleaf (University of Rochester).

Virtual Hybrid Edge Detection

Abstract : Hybrid imaging is based on combining a high contrast/low resolution technique, often modeled by an elliptic PDE, with a low contrast/high resolution technique, modeled by a hyperbolic equation. Coupling provided by an interaction between the underlying physics of these two waves, such as the photo acoustic effect, allows one to obtain both high contrast and high resolution. We describe a new kind of "hybrid" imaging which is in fact based on only one physical phenomenon (EIT), with the high resolution waves being provided by formal manipulation through the introduction of a virtual variable. This is joint work with Matti Lassas, Matteo Santacesaria, Samuli Siltanen and Gunther Uhlmann.

Boaz Haberman (University of California, Berkeley).

Unique continuation and inverse boundary value problems for elliptic operators with unbounded gradient terms

Abstract : This talk will focus on unique continuation and the inverse boundary value problem for operators of the form $D^2 + A.D + q$. Both of these problems involve Carleman estimates for the Laplacian. When the coefficients A and q are sufficiently singular, the necessary Carleman estimates are no longer true, and some additional idea is needed to overcome this difficulty. We will discuss Wolff's work on the unique continuation problem and describe some recent work on the inverse boundary value problem.

Hiroshi Isozaki (Tsukuba University).

Inverse scattering on non-compact manifolds with general metric

Abstract : We consider the inverse scattering on non-compact Riemannian manifolds (or orbifolds) $\mathcal{M}$ consisting of a union of open sets :

$$\mathcal{M} = \mathcal{K} \cup \mathcal{M}_1 \cup \cdots \mathcal{M}_{N+N'},$$

where $\overline{\mathcal{K}}$ is compact, $\mathcal{M}_i$ is diffeomorphic to $(1, \infty) \times M_i$, M_i being a compact $n-1$ dimensional manifold (or orbifold) endowed with the metric h_{M_i}. On each end $\mathcal{M}_i$, the metric of $\mathcal{M}$ is assumed to behave like

$$ds^2 \sim (dr)^2 + \rho_i(r)^2 h_{M_i}, \quad r \to \infty,$$

and $\rho_i(r)$ has the form

$$\rho_i(r) = A \exp\big(c_0 r + \frac{\beta}{1-\alpha} r^{-\alpha}\big), \quad 0 < \alpha < 1, \quad \text{or} \quad A \exp\big(c_0 r\big) r^{\beta}.$$

We assume that for $1 \leq i \leq N$, ends $\mathcal{M}_i$ have regular infinities, i.e. we assume that $c_0 \geq 0$ and if $c_0 = 0$, then $\beta > 0$, while for $N+1 \leq i \leq N+N'$, they have cusps, i.e. we assume $c_0 \leq 0$ and if $c_0 = 0$, then $\beta < 0$. So, it includes typical examples such as regular infinity of hyperbolic manifolds, $\rho(r) = e^r$; euclidean end, $\rho(r) = r$; cusp of hyperbolic manifolds, $\rho(r) = e^{-r}$.

By observing the asymptotic behavior at infinity of generalized eigenfunctions for the Laplace operator on $\mathcal{M}$, we introduce the S-matrix (physical S-matrix for regular ends and generalized S-matrix for cusps), and then solve the inverse scattering problem, i.e. the recovery of the manifold $\mathcal{M}$ from one component of the S-matrix (for all energies).

This is a joint work with Y. Kurylev and M. Lassas.

Alexandre Jollivet (Université de Lille).

Inverse scattering at high energies for classical particles in a long range electromagnetic field

Abstract : We consider in $\mathbf{R}^n$, $n \geq 2$, a classical relativistic particle moving in a smooth and long range static electromagnetic field F decomposed as a sum of a known long range tail F^l and an unknown short range part F^s. We define a scattering map and we give estimates and asymptotics for this map in the regime of small scattering angles compared to the dynamics generated by F^l. We show, in particular, that at high energies the first component of the scattering map uniquely determines the x-ray transform of F^s on a big enough set of lines so that it uniquely determines F^s. Similar results for the nonrelativistic Newton equation will also be discussed. Finally we will briefly recall uniqueness results for the inverse scattering problem at fixed energy.

Yavar Kian (Université d'Aix-Marseille).

A multidimensional Borg-Levinson theorem for magnetic Schrödinger operators with partial spectral data

Abstract : Let $\mathcal{A}$ be the Dirichlet realization of the magnetic Schrödinger operator $(-\mathrm{i}\nabla + A)^2 + V$ on a bounded domain $\Omega \subset \mathbf{R}^n$, $n \geq 2$, with $A \in W^{1,\infty}(\Omega, \mathbf{R}^n)$ and $V \in L^\infty(\Omega)$. We associate to $\mathcal{A}$ the full boundary spectral data given by the set $\{(\lambda_k, \partial_\nu \phi_{k|\partial\Omega}) : \ k \geq 1\}$, where $\{\lambda_k : \ k \in \mathbf{N}^*\}$ is the non-decreasing sequence of eigenvalues of $\mathcal{A}$, $\{\phi_k : \ k \in \mathbf{N}^*\}$ an associated Hilbertian basis of eigenfunctions and ν the unit outward normal vector to $\partial\Omega$. We will consider the multidimensional Borg-Levinson theorem of determining both the magnetic field dA and the electric potential V from partial knowledge of the boundary spectral data of $\mathcal{A}$.

Evgeny Lakshtanov (University of Aveiro).

On some constructive and explicit solutions in inverse scattering theory

Abstract : First I will talk about recent constructions of bodies invisible in frames of geometric optics. Corresponding rigorous results in the high frequency regime will be presented. Then I will discuss some constructive methods of potential/obstacle reconstruction and will give a short review of Interior Transmission Eigenvalues. These eigenvalues play the same role in transmission scattering problems as Laplacian spectrum in the scattering by an obstacle. Finally, I will present some our recent explicit formula for potential reconstruction. Results mentioned above were obtained in collaboration with R.Novikov, B.Sleeman, B.Vainberg.

Hongyu Liu (Hong Kong Baptist University).

Stable determination of polyhedral scatterers by a minimal number of far-field measurements

Abstract : In this talk, the speaker will present two stability results of logarithmic type in determining conducting scatterers by a minimal number of electromagnetic (EM) far-field measurements. The admissible Lipschitz scatterers are of general polyhedral type, which may consist of finitely many (with an unknown number) solid polyhedra and screen-type cells. They satisfy minimum regularity assumptions. For the general case, the stability estimate is established by using two EM far-field measurements. If there are no screen-type components presented, then the stability estimate is established by using only a single far-field measurement.

Georgi Popov (Université de Nantes).

Isospectral deformations K.A.M. tori and spectral rigidity

Abstract : We are interested in the spectral rigidity of the Laplace-Beltrami operator in the case when the corresponding classical Hamiltonian system is either completely integrable or close to a non-degenerate completely integrable system. By the Kolmogorov-Arnold-Moser (K.A.M.) theorem there exists a large family of invariant tori of the classical system. We show that the values of the corresponding Mather's beta function given by the average action on the K.A.M. tori is an isospectral invariant. As an application we obtain infinitesimal rigidity of Liouville billiard tables in dimensions two and three. Spectral rigidity is obtained as well in the presence of elliptic bouncing ball geodesics. The proof is based on a construction of smooth with respect to the parameter of the deformation families of quasi-modes associated with the K.A.M. tori.

Rakesh (University of Delaware)

The inverse backscattering problem

Abstract : We discuss the, still unsolved, problem of recovering a smooth, compactly supported potential on R^3 from its backscattering data. We describe the known results and our partial result regarding the recovery of angularly controlled potentials and we state some simple sub-problems which are also unsolved. This is based on joint work with Gunther Uhlmann.

Alberto Ruiz (Universidad Autónoma de Madrid).

Stability of the Calderón problem with partial data

Abstract : We prove a log-log stability of Calderón's Inverse problem with the set of data considered by Kenig, Sjöstrand and Uhlmann by recovering a double Fourier and geodesic transform in polar coordinates. This is a joint work with D.

dos Santos Ferreira and P. Caro.

Antonio Sà Barreto (Purdue University)

Unique continuation from infinity and inverse problems in geometric scattering

Abstract : We will discuss unique continuation for the radiation fields in some cases of geometric scattering and we will show how this can be used to recover an asymptotically hyperbolic manifold from the knowledge of the scattering operator restricted to a subset of the boundary at infinity.

Éric Soccorsi (Université d'Aix Marseille).

An inverse spectral problem in a periodic quantum waveguide

Abstract : Given an infinite cylindrical domain $\Omega = \omega \times \mathbf{R}$, where ω is a bounded $C^{1,1}$ domain of $\mathbf{R}^2$, we consider the operator $A = -\Delta + q$, acting in $L^2(\Omega)$, with Dirichlet boundary conditions. Here, $q \in L^\infty(\Omega)$ is real-valued and 1-periodic with respect to the first variable of Ω. The operator A admits a direct fiber decomposition A_θ, $\theta \in [0, 2\pi)$. We prove uniqueness and stability in the inverse problem of determining the electric potential q from the asymptotic behaviour of the spectral data of any fiber A_θ, for θ in $[0, 2\pi)$.

This is joint work with Otared Kavian (Versailles) and Yavar Kian (Marseille).

John Sylvester (University of Washington).

The Role of Evanescence in the Inverse Source Problem

Abstract : The inverse source problem for the Helmholtz equation seeks to recover information about a radiating source from remote observations of a monochromatic (single frequency) radiated wave measured far from the source (the far field). The physical phenomenon of evanescence, which limits imaging resolution to the size of a wavelength, plays a major role in both the inverse source and inverse scattering problem. I will explain how the subspace of non-evanescent far fields encodes information about the size and location (support) of a source or scatterer, and discuss the properties of these subspaces that relate to our ability to distinguish between fields radiated or scattered from different locations.

Gerald Teschl (University of Vienna).

Peakon asymptotics for the dispersionless Camassa-Holm equation

Abstract : We discuss direct and inverse spectral theory for the isospectral problem of the dispersionless Camassa-Holm equation, where the weight is allowed to be a finite signed measure. In particular, we prove that this weight is uniquely determined by the spectral data and solve the inverse spectral problem for the class of measures which are sign definite. The results are applied to deduce several facts for the dispersionless Camassa-Holm equation. In particular, we show that initial conditions with integrable momentum asymptotically split into a sum of peakons as conjectured by McKean.

Andras Vasy (Stanford University).

The local inverse problem for the geodesic X-ray transform on tensors and boundary rigidity

Abstract : In this talk, based on joint work with Plamen Stefanov and Gunther Uhlmann, I discuss the geodesic X-ray transform on a Riemannian manifold with boundary. The geodesic X-ray transform on functions associates to a function its integral along geodesic curves, so for instance in domains in Euclidean space along straight lines. The X-ray transform on symmetric tensors is similar, but one integrates the tensor contracted with the tangent vector of the geodesics. I will explain how, under a convexity assumption on the boundary, one can invert the local geodesic X-ray transform on functions, i.e. determine the function from its X-ray transform, in a stable manner. I will also explain how the analogous result can be achieved on one forms and 2-tensors up to the natural obstacle, namely potential tensors (forms which are differentials of functions, respectively tensors which are symmetric gradients of one-forms).

Here the local transform means that one would like to recover a function (or tensor) in a suitable neighborhood of a point on the boundary of the manifold given its integral along geodesic segments that stay in this neighborhood (i.e. with both endpoints on the boundary of the manifold). Our method relies on microlocal analysis, in a form that was introduced by Melrose.

I will then also explain how, under the assumption of the existence of a strictly convex family of hypersurfaces foliating the manifold, this gives immediately the solution of the global inverse problem by a stable 'layer stripping' type construction. Finally, I will discuss the relationship with, and implications for, the boundary rigidity problem, i.e. determining a Riemannian metric from the restriction of its distance function to the boundary.

Ting Zhou (Northeastern University).

On uniqueness of an inverse problem for the time-harmonic Maxwell equations

Abstract : The inverse boundary value problem for the time-harmonic Maxwell equations is a nonlinear problem to determine electromagnetic parameters

of the medium, namely the magnetic permeability, the electric permittivity and the conductivity, on a bounded domain using the measurements of the electromagnetic fields on the boundary of the domain. I will present both the boundary uniqueness and interior uniqueness of the parameters, where we assume that the unknown parameters are described by continuously differentiable functions. The key ingredient in proving the uniqueness is the complex geometrical optics (CGO) solutions.

This is a joint work with Pedro Caro.

Miren Zubeldia (BCAM - Basque Center for Applied Mathematics, Spain and University of Helsinki, Finland).

Transmission eigenvalues for magnetic Schrödinger operator

Abstract : The transmission eigenvalue problem is a non-selfadjoint and nonlinear eigenvalue problem that is not covered by the standard theory of eigenvalue problems for elliptic operators. It appears in the study of the inverse scattering theory ; transmission eigenvalues provide information about material properties of the scattering media. Most of the work on transmission eigenvalues has so far been for second order operators and zeroeth order perturbations.

In this talk we present some results of a joint work with A. García and E. Vesalainen where we extend the theory of transmission eigenvalues for higher-order main terms. In particular, we focus on the discreteness of transmission eigenvalues for magnetic Schrödinger operators, using Sylvester's approach via upper triangular compact operators.