Conference

"GEOMETRIC INVERSE PROBLEMS"

IHP, June 08 – 12

Organizers : Colin Guillarmou, Alexandre Jollivet and Mikko Salo

Scientific Committee : Matti Lassas, Yaroslav Kurylev, Gabriel Paternain, Todd Quinto

PROGRAM

Monday, June 8th

9 :30 – 10 :30 : **Plamen Stefanov** (Purdue University)
The geodesic ray transform on Riemannian surfaces with conjugate points

10 :30 – 11 :00 : Coffee, tea break

11 :00 – 12 :00 : **Haomin Wen** (University of Pennsylvania)
Lens rigidity and scattering rigidity in two dimensions

12 :00 – 14 :00 : *Lunch*

14 :00 – 15 :00 : **Gabriel Paternain** (University of Cambridge)
Recovering a connection from parallel transport along geodesics

15 :00 – 16 :00 : **Sean Holman** (University of Manchester)
On the stability of the geodesic ray transform

16 :00 – 16 :30 : Coffee, tea break

16 :30 – 17 :30 : **Francis Chung** (University of Michigan)
Inverse Problems for the Hodge Laplacian

Tuesday, June 9th

9 :00 – 10 :00 : **Sergei Ivanov** (Steklov Institute of Mathematics at St. Petersburg)
Perturbations of boundary distance functions and scattering maps

10 :00 – 10 :30 : Coffee, tea break

10 :30 – 11 :30 : **François Monard** (University of Washington)
Explicit inversions of ray transforms over simple surfaces

11 :30 – 12 :30 : **Hanming Zhou** (University of Washington)
Invariant distributions and tensor tomography for Gaussian thermostats

12 :30 – 14 :30 : *Lunch*

14 :30 – 15 :30 : **Gunther Uhlmann** (University of Washington)
Boundary Rigidity and Lens Rigidity : Progress and Challenges

15 :30 – 16 :00 : Coffee, tea break

16 :00 – 17 :00 : **Joonas Ilmavirta** (University of Jyväskylä)
Radon transforms on groups

Wednesday, June 10th

9 :00 – 10 :00 : **Vladimir Sharafutdinov** (Novosibirsk State University)
Killing tensor fields on the 2-torus

10 :00 – 10 :30 : Coffee, tea break

10 :30 – 11 :30 : **Lauri Oksanen** (University College London)
The X-ray transform restricted on light rays and cosmic microwave background

11 :30 – 12 :30 : **Yang Yang** (Purdue University)
A Traveltime Inverse Problem in Spacetime

12 :30 – 14 :30 : *Lunch*

14 :30 – 15 :30 : **Jan Boman** (Stockholm University)
Tomographic region-of-interest reconstruction from incomplete data

15 :30 – 16 :00 : Coffee, tea break

16 :00 – 17 :00 : **Hai Zhang** (École Normale Supérieure)
Stability of recovering wave-speed from boundary measurements

18 :30 :

COCKTAIL RECEPTION

Thursday, June 11th

9 :00 – 10 :00 : **Simon Gindikin** (Rutgers University)
Spectral decomposition of some ultrahyperbolic equations

10 :00 – 10 :30 : Coffee, tea break

10 :30 – 11 :30 : **Linh Nguyen** (University of Idaho)
On the streak artifacts in limited angle computed X-ray tomography

11 :30 – 12 :30 : **Katya Krupchyk** (University of California, Irvine)
L^p bounds on eigenfunctions for operators with double characteristics

12 :30 – 14 :30 : *Lunch*

14 :30 – 15 :30 : **Raluca Felea** (Rochester Institute of Technology)
Fourier Integral Operators with cusp singularities

15 :30 – 16 :00 : Coffee, tea break

Friday, June 12th

9 :00 – 10 :00 : **Maciej Zworski** (University of California, Berkeley)
Heat traces and inverse problems for resonances

10 :00 – 10 :30 : Coffee, tea break

10 :30 – 11 :30 : **Daniel Faraco** (Universidad Autonóma de Madrid)
Obstruction to the existence of Limiting Carleman Weights

Abstracts

Jan Boman (Stockholm University)
Tomographic region-of-interest reconstruction from incomplete data
Abstract : In a series of articles Rolf Clackdoyle, Fredric Noo, Michel Defrise, and others have showed that stable reconstruction in a proper subset — the region of interest — of the support of the unknown function can be done from incomplete data in some cases. Very little is known about the general problem of which sets of data are needed for stable reconstruction in a given region of interest. I will describe some of the results of Clackdoyle and coworkers and make some remarks on the general problem.

Francis Chung (University of Michigan)
Inverse Problems for the Hodge Laplacian
Abstract : In this talk I will discuss a Calderon-type inverse problem for the Hodge Laplacian on manifolds. I will motivate the problem by explaining the connection to inverse problems for the Maxwell equations, and talk about some of the ideas involved in solving such a problem. In particular, I will describe the connection between tensor tomography and the inverse problem for the Hodge Laplacian. This is joint work with Mikko Salo and Leo Tzou.

Daniel Faraco (Universidad Autonóma de Madrid)
Obstruction to the existence of Limiting Carleman Weights
Abstract : In a seminal paper Dos Santos Ferreira, Kenig, Salo and Uhlmann proved that Calderón inverse problem in a Riemannian manifold can be solved if it admits a conformal parallel vector field. In this talk I will relate this condition with the Weyl and Cotton York tensor. As a corollary we can analyze classical homogeneous manifolds and extend a result of Liimatainen and Salo showing that the set of bad metrics, in the sense that this method for solving Calderón problem is not working is open and dense. This is a joint work with P. Angulo, L. Guijarro and A.Ruiz.

Raluca Felea (Rochester Institute of Technology)
Fourier Integral Operators with cusp singularities
Abstract : We consider Fourier Integral Operators with one sided cusp singularities which appear in inverse problems related to seismology and radar imaging and also in integral geometry as generalized Radon transforms. We use microlocal analysis to describe the composition calculus of these operators and we show that they have the wave front relation in the union of the diagonal and another singular lagrangian.

Simon Gindikin (Rutgers University)
Spectral decomposition of some ultrahyperbolic equations
Abstract : We develop spectral theory of the operators Laplace-Beltrami on hyperboloids of arbitrary signature considering horospherical waves. We show that the continuos spectrum corresponds to the real boundary but the discrete spectrum is connected with the complex boundary and cohomological boundary values.

Sean Holman (University of Manchester)
On the stability of the geodesic ray transform
Abstract : In this talk I will discuss stability estimates for inversion of the geodesic X-ray transform on a non-trapping manifold with boundary. In the case of a simple manifold such estimates have been known for a long time, but for the more general non-trapping case, which allows the presence of conjugate points, less is known. I will review recent attempts to establish such estimates via characterisation of the normal operator as a sum of Fourier integral operators, and highlight the geometric issues arise.

Joonas Ilmavirta (University of Jyväskylä)
Radon transforms on groups
Abstract : Can one recover a function on a closed manifold from its integrals over all periodic geodesics ? We study this problem when the manifold is a Lie group. We present a relatively simple method to characterize the Lie groups on which this recovery is possible. The method works with low regularity (distributions) and also tensor fields (on tori). If time permits, we will also discuss an analogous problem on finite groups.

Sergei Ivanov (Steklov Institute of Mathematics at St. Petersburg)
Perturbations of boundary distance functions and scattering maps
Abstract : A simple Riemannian or Finsler metric on a disc determines the boundary distance function : it is the restriction of the geodesic distance to the boundary. A related structure is the scattering map which sends incoming velocity vectors of geodesics to outgoing ones. Which functions can be realized as boundary distance functions of Riemannian metrics is a widely open question. We solve a local variant of this question for Finsler metrics. Namely we show that every sufficiently small perturbation of the boundary distance function is again a boundary distance function of some Finsler metric. A similar result holds for scattering maps and symplectic perturbations. Some applications of the construction are interesting examples of dynamical systems, volume comparison for Riemannian metrics, and a new proof of injectivity of the geodesic ray transform.

Katya Krupchyk (University of California, Irvine)
L^p bounds on eigenfunctions for operators with double characteristics
Abstract : We discuss sharp L^p bounds on the ground states for a class of semiclassical pseudodifferential operators with double characteristics and complex valued symbols, under the assumption that the quadratic approximations along the double characteristics are elliptic. This is a joint work with Gunther Uhlmann.

François Monard (University of Washington)
Explicit inversions of ray transforms over simple surfaces
Abstract : We will first review reconstruction formulas for the geodesic X-ray transform for functions and solenoidal vector fields, first derived by Pestov and Uhlmann (IMRN 04), based on studying the interplay between geodesic flow and Hilbert transform. We will review recent generalizations of this approach by the author and will go over how far such ideas can generalize.

We will then focus on the attenuated geodesic X-ray transform, including reconstruction algorithms for functions and vector fields (attenuated Doppler transform),

as well as some numerical implementations.

Linh Nguyen (University of Idaho)
On the streak artifacts in limited angle computed X-ray tomography
Abstract : In this talk, we consider the limited angle problem in computed x-ray tomography. A common practice is to use filtered back-projection formulas with the limited data for the reconstruction. This practice reconstructs visible singularities of the imaged function. However, it also generates artifacts along some straight lines. We will characterize the strength of these artifacts.

Lauri Oksanen (University College London)
The X-ray transform restricted on light rays and cosmic microwave background
Abstract : We consider a mathematical model for measurements of cosmic microwave background radiation and the inverse problem to determine the spacetime from these measurements. We linearize the problem around the Einstein-de Sitter model, and show that this leads to a tensor tomography problem along the light rays, that is, we obtain a limited angle X-ray transform of the perturbation of the metric tensor restricted on the light rays of the background metric. We analyze what type of singularities can be reconstructed from this restricted X-ray transform. This is a joint work with M. Lassas, P. Stefanov and G. Uhlmann.

Gabriel Paternain (University of Cambridge)
Recovering a connection from parallel transport along geodesics
Abstract : I will discuss the inverse problem of recovering a unitary connection from the parallel transport along geodesics of a compact Riemannian manifold of negative curvature and strictly convex boundary. The solution to this geometric inverse problem is based on a range of techniques, including energy estimates and regularity results for the transport equation associated with the geodesic flow. This is joint work with Colin Guillarmou, Mikko Salo and Gunther Uhlmann.

Vladimir Sharafutdinov (Novosibirsk State University)
Killing tensor fields on the 2-torus
Abstract : A symmetric tensor field on a Riemannian manifold is called Killing field if the symmetric part of its covariant derivative is equal to zero. There is a one to one correspondence between Killing tensor fields and first integrals of the geodesic flow which depend polynomially on the velocity. Therefore Killing tensor fields closely relate to the problem of integrability of geodesic flows. In particular, the following question is still open : does there exist a Riemannian metric on the 2-torus which admits an irreducible Killing tensor field of rank ≥ 3 ? We obtain two necessary conditions on a Riemannian metric on the 2-torus for the existence of Killing tensor fields. The first condition is valid for Killing tensor fields of arbitrary rank and relates to closed geodesics. The second condition is obtained for rank 3 Killing tensor fields and relates to isolines of the Gaussian curvature.

Plamen Stefanov (Purdue University)
The geodesic ray transform on Riemannian surfaces with conjugate points
Abstract : We study the geodesic X-ray transform X on compact Riemannian surfaces with conjugate points. Regardless of the type of the conjugate points, we show that we cannot recover the singularities and therefore, this transform is always

unstable. We describe the microlocal kernel of X and relate it to the conjugate locus. We present numerical examples illustrating the cancellation of singularities. We also show that an attenuation stabilizes the transform, if there are no more than two conjugate points on each geodesic.

Gunther Uhlmann (University of Washington)
Boundary Rigidity and Lens Rigidity : Progress and Challenges

Haomin Wen (University of Pennsylvania)
Lens rigidity and scattering rigidity in two dimensions
Abstract : Scattering rigidity of a Riemannian manifold allows one to tell the metric of a manifold with boundary by looking at the directions of geodesics at the boundary. Lens rigidity allows one to tell the metric of a manifold with boundary from the same information plus the length of geodesics. There are a variety of results about lens rigidity but very little is known for scattering rigidity. We will discuss the subtle difference between these two types of rigidities and prove that they are equivalent for a large class of two-dimensional Riemannian. In particular, two-dimensional simple Riemannian manifolds (such as the flat disk) are scattering rigid since they are lens/boundary rigid (Pestov–Uhlmann, 2005).

Yang Yang (Purdue University)
A Traveltime Inverse Problem in Spacetime
Abstract : We consider an inverse problem in a Lorentzian manifold (M, g). We show that the time measurements, which is the knowledge of the Lorentzian separation function on a submanifold Σ, determine the derivatives of the metric tensor. This result can be used to study the global determination of a spacetime if it either has a real-analytic structure or is stationary and satisfies the Einstein-scalar field equations. The presented results are Lorentzian counterparts of the extensively studied inverse problems in Riemannian geometry – determination of the jet of a metric and the boundary rigidity problem. This is a joint work with M. Lassas and L. Oksanen.

Hai Zhang (École Normale Supérieure)
Stability of recovering wave-speed from boundary measurements
Abstract : We report recent progress on the stability of recovering wave speed from boundary measurements which are motivated from problems in geophysics. Two type of measurements are considered : one is the scattering relation and the other is the Dynamic Dirichlet to Neumman map. In both cases, we derive stability estimates for a class of wave speeds which correspond to non-simple Riemannian metrics.

Hanming Zhou (University of Washington)
Invariant distributions and tensor tomography for Gaussian thermostats
Abstract : In this talk we consider the Gaussian thermostat ray transform on both closed Riemannian surfaces and compact Riemannian surfaces with boundary. We establish certain results on the injectivity of the thermostat ray transform and the surjectivity of its adjoint. This is a joint work with Yernat Assylbekov.

Maciej Zworski (University of California, Berkeley)
Heat traces and inverse problems for resonances
Abstract : Suppose that V is a bounded compactly supported potential in odd dimensions. Then solutions of $(\partial_t^2 - \Delta_x + V(x))u = 0$ with compactly supported initial data have expansions valid for x in compact sets :

$$u(t,x) = \sum_{j=1}^{N} e^{-it\lambda_j} u_j(x) + O(e^{-At}), \operatorname{Im}\lambda_j \geq -A.$$

These λ_j's are called scattering resonances and replace eigenvalues of problems on open domains.

It has been known for a while that for a real valued smooth V there are infinitely many resonances (Sá Barreto–Z) and that generic uniformly bounded (or smooth) V have resonances saturating the upper bounds on their counting functions (Christiansen–Hislop).

In joint work with Hart Smith we show that any bounded compactly supported potential has to have some resonances. That is done by proving the following (seemingly unknown) result : for V uniformly bounded,

$$t^{\frac{n}{2}} \operatorname{tr}(e^{-t(-\Delta+V)} - e^{t\Delta}) \in C^\infty([0,\infty)) \Longrightarrow V \in C^\infty.$$

In fact, we show that regularity of the potential follows from some information about resonances which is a very modest inverse result.